

# Practical Issues in the Bayesian Design of Early Clinical Studies of Treatment Efficacy

Phil Woodward, Frances Hackman,  
Phil Stanley  
Pfizer, UK

# Summary

- Study Design
- Prior Knowledge
- Decision Criteria
- Study Operating Characteristics
- Interim Analyses
- Issues & Value of Approach

# “Proof of Concept” Study

- Novel treatment for Osteoarthritic pain
  - Test compound
  - Positive control (Standard of Care)
  - Negative control (Placebo)
- Response is patient reported pain score
- Design options
  - Parallel groups, 2 weeks duration
  - Cross-over, 2 x 2 weeks + washout

# Prior Knowledge

## Historical Studies (OA population)

study	treatment	N	CFB.LSmean	se.CFB	residual std (CFB)	baseline mean	scale
1	Placebo	201	-0.7	0.23	3.26	10.8	WOMAC(0-20)
1	Naproxen	198	-2.7	0.23	3.24	11.0	WOMAC(0-20)
2	Placebo	242	-1.2	0.21	3.27	10.5	WOMAC(0-20)
2	Naproxen	226	-2.8	0.21	3.16	11.1	WOMAC(0-20)
4	Placebo	66	-2.6	0.47	3.82		WOMAC(0-20)
4	Naproxen	69	-4.6	0.46	3.82		WOMAC(0-20)
6	Placebo	41	-2.3	0.58	3.70	11.5	WOMAC(0-20)
6	Naproxen	39	-4.4	0.59	3.70	11.5	WOMAC(0-20)
7	Placebo	205	-2.2	0.25	3.57	10.9	WOMAC(0-20)
7	Naproxen	204	-3.3	0.25	3.57	10.7	WOMAC(0-20)
9	Placebo	110	-2.8	0.36	3.82	11.3	WOMAC(0-20)
11	Placebo	42	-1.0	0.50	3.21		WOMAC(0-20)
3	Placebo	217	-0.7	0.21	3.09	10.6	WOMAC(0-20)
3	Naproxen	207	-2.9	0.21	3.02	10.5	WOMAC(0-20)
5	Placebo	73	-3.2	0.47	4.02	13.8	Scaled from VAS (0-100)
8	Placebo	82	-0.4	0.44	4.03	10.5	Scaled from VAS (0-500)
8	Naproxen	75	-3.5	0.47	4.03	10.4	Scaled from VAS (0-500)
10	Placebo	117	-0.9	0.32	3.48		WOMAC(0-20)
10	Naproxen	118	-3.1	0.32	3.48		WOMAC(0-20)
13	Placebo	104	-4.4		4.50	13.2	Scaled from VAS (0-100)
13	Naproxen	117	-7.0		4.50	13.0	Scaled from VAS (0-100)

# Prior Distributions from Historical Data

- Normal Linear Mixed Model

$$Y_{ij} = \mu + \alpha_i + s_j + (\alpha s)_{ij} + e_{ij}$$

$$s_j \sim N(0, \omega_s^2)$$

$$(\alpha s)_{ij} \sim N(0, \omega_{\alpha s}^2)$$

$$e_{ij} \sim N(0, SE_{ij}^2)$$

$Y_{ij}$  are the observed treatment means for the  $i^{\text{th}}$  treatment in the  $j^{\text{th}}$  study, and

$SE_{ij}$  are their associated standard errors

# Prior Distributions from Historical Data

Model fitted  
using WinBUGS  
via GUI BugsXLA

BugsXLA removes  
need for WinBUGS  
expertise

	Label	Mean	St.Dev.	2.5%	Median	97.5%	WinBUGS Name
	CONSTANT	-1.8670	0.3951	-2.7130	-1.8480	-1.1350	Beta0
treatment	Placebo	0.0000	0.0000				X.Eff[1,1]
treatment	Naproxen	-1.6730	0.3207	-2.3610	-1.6580	-1.0770	X.Eff[1,2]
	SD(study)	0.8881	0.3994	0.2474	0.8259	1.8390	sigma.Z[1]
	SD(study x treatment)	0.3248	0.2526	0.0131	0.2671	0.9668	sigma.Z[2]

<b>Model</b>	[Prior Data!\$B\$1:\$F\$13]
Distribution	Normal
Link	Identity
Response	CFB.LSmean;se.CFB
Fixed	treatment
Random	study+study:treatment

<b>Priors</b>	
CONSTANT	N(mu=-2.55, sigma=121)
treatment	N(mu=0, sigma=121)
study	Norm(0,tau^2); tau ~ Half-N(sigma=2)
study x treatment	Norm(0,tau^2); tau ~ Half-N(sigma=1)

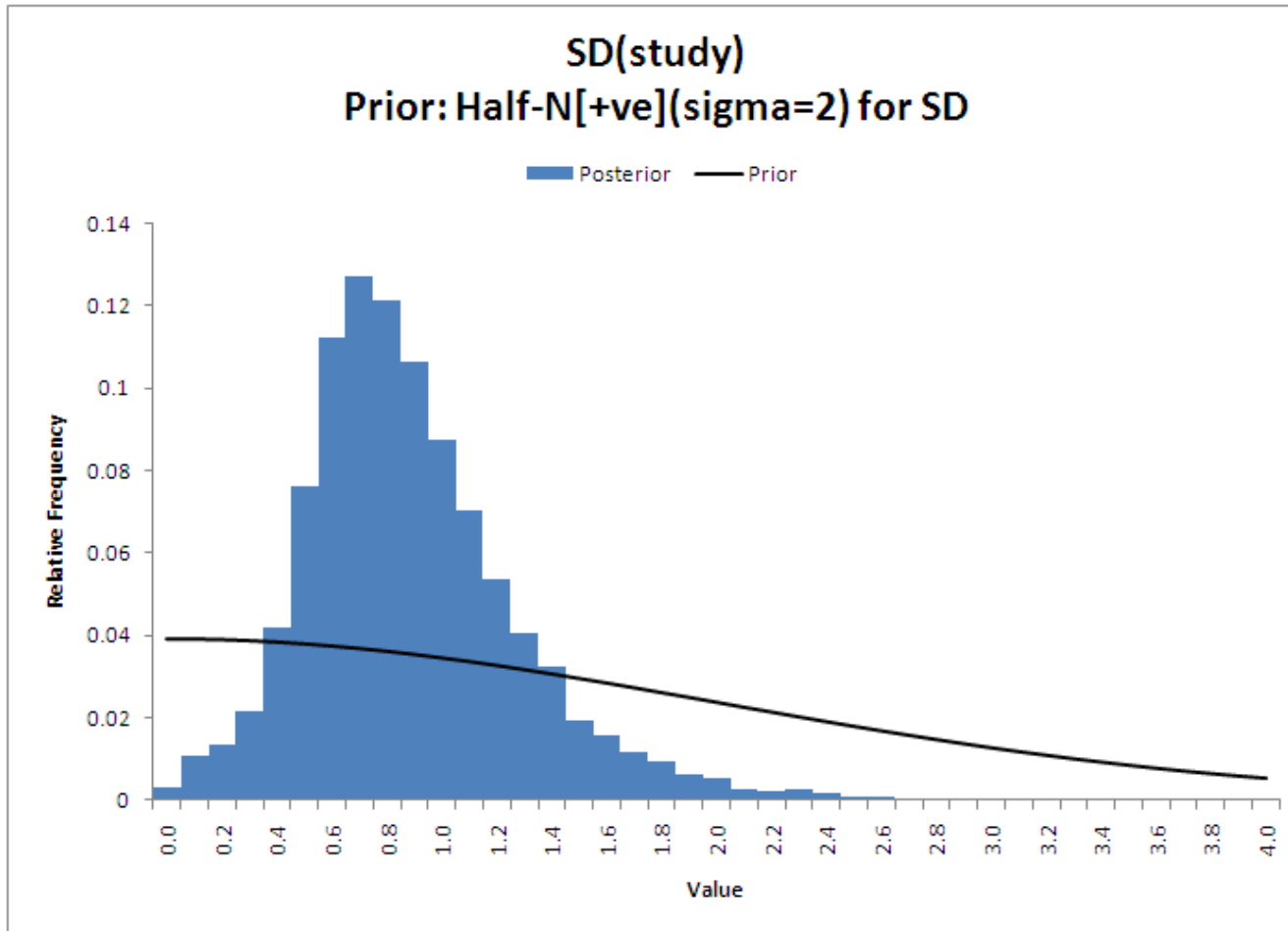
<b>WinBUGS MCMC Settings</b>	
Burn-In:	5000
Samples:	10000 (Thin:1; Chains:1)
Run took	15 seconds
BugsXLA (Alpha 5.0) 2009.Nov.21.(17.25)	

BugsXLA is provided without any warranty of any kind. eith  
The user is responsible for any consequences arising from  
See also the disclaimer provided with the WinBUGS packa

The screenshot shows the 'Bayesian Model Specification' dialog box. It includes sections for 'Data' (range: 'Prior Data!\$B\$1:\$F\$13'), 'Model' (Distribution: Normal, Link: Identity, Response: CFB.LSmean;se.CFB), 'Factors' (Fixed: treatment, Random: study+study:treatment), and 'Covariates' (Independent and Random Coeffs). There are also checkboxes for 'Names in first row', 'Blank cells = MVs (Response only)', 'Non-Linear Model', 'Longitudinal Model', and 'Predictions or Contrasts'.

# Visual Comparison of Prior & Posterior



Posterior Percentiles								
0.1%	0.5%	1.0%	2.5%	5%	10%	20%	25%	50%
0.0	0.1	0.1	0.2	0.4	0.5	0.6	0.6	0.8
99.9%	99.5%	99.0%	97.5%	95%	90%	80%	75%	
3.1	2.3	2.1	1.8	1.6	1.4	1.2	1.1	

# Prior Distributions from Historical Data

- Prior for treatment mean responses in future study is predictive distribution not posterior
- Predictive distribution obtained via WinBUGS
  - (Std of Care, Pbo) has a bivariate prior dist.
    - Due to between study variance
  - Multi-Variate Normal fitted
- Study designs assessed via simulation
  - Use “well known” NLM result to derive posterior
  - Analytical result speeds up simulation

## Prior Distributions from Historical Data

*Lemma:* Lindley and Smith (JRSSB,1972,34,1-18)

If  $\theta_1$  a vector of  $p_1$  parameters

$$y \sim N(A_1\theta_1, C_1),$$

and  $\theta_2$  a vector of  $p_2$  hyperparameters

$$\theta_1 \sim N(A_2\theta_2, C_2),$$

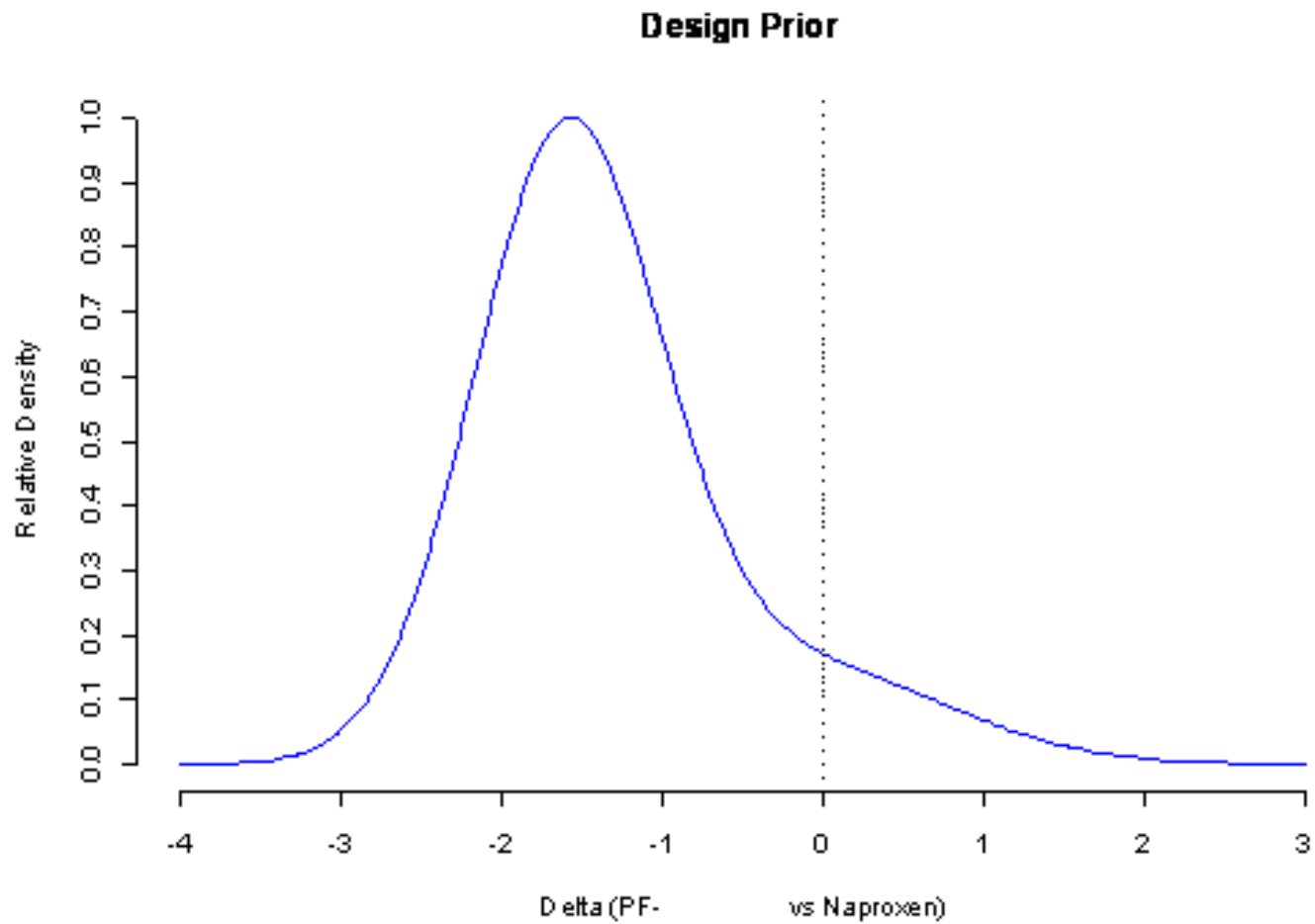
then  $p(\theta_1 | y)$  is  $N(Bb, B)$  with

$$B^{-1} = A'_1 C_1^{-1} A_1 + C_2^{-1}$$

and

$$b = A'_1 C_1^{-1} y + C_2^{-1} A_2 \theta_2$$

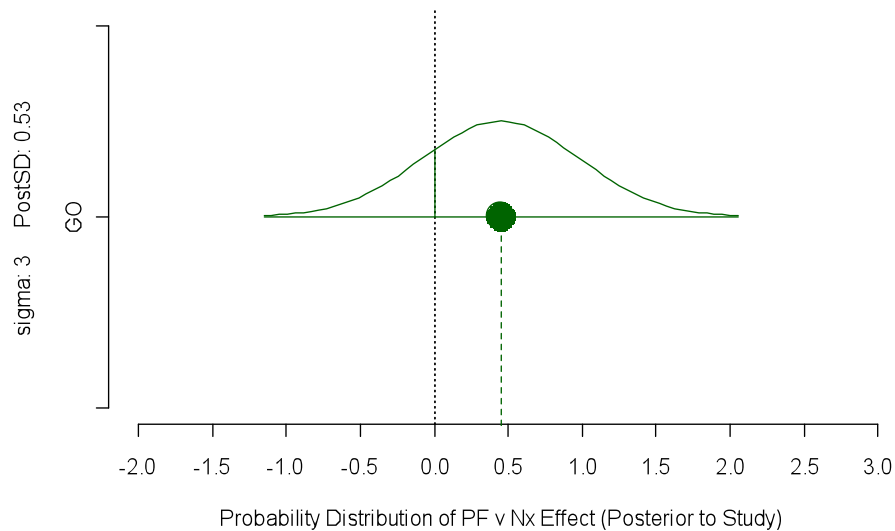
# Elicited Prior Distribution (Novel Treatment)



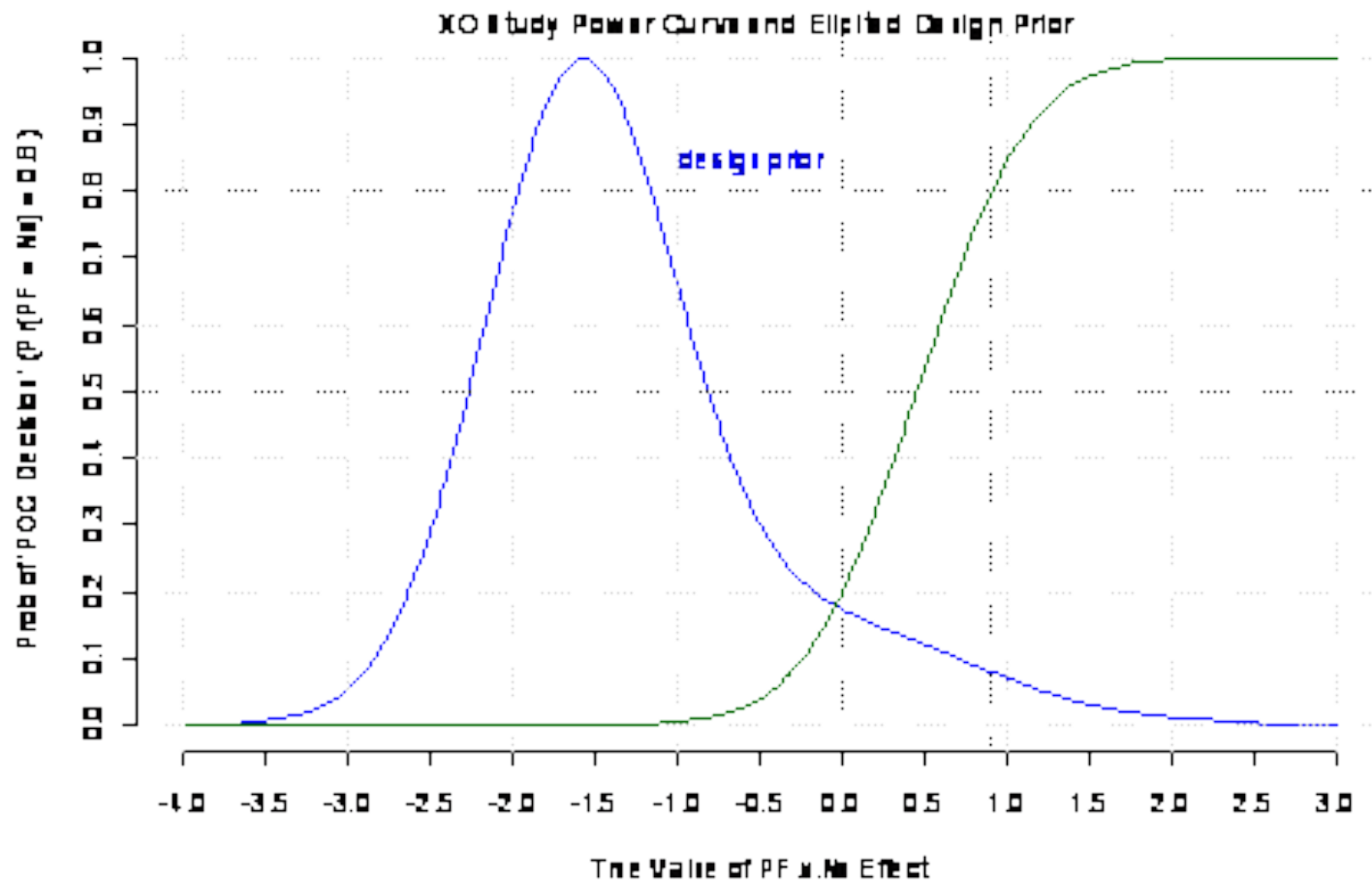
# Decision Criteria

- C1: At least 90% sure better than placebo.
- C2: At least 80% sure better than naproxen.

Decision Criteria: Minimum Evidence Required to GO



# Study OC (C2) with Prior Overlaid

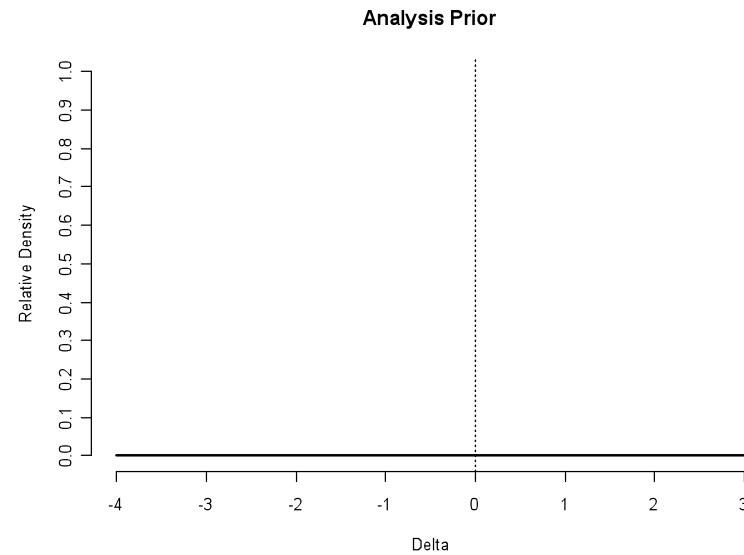
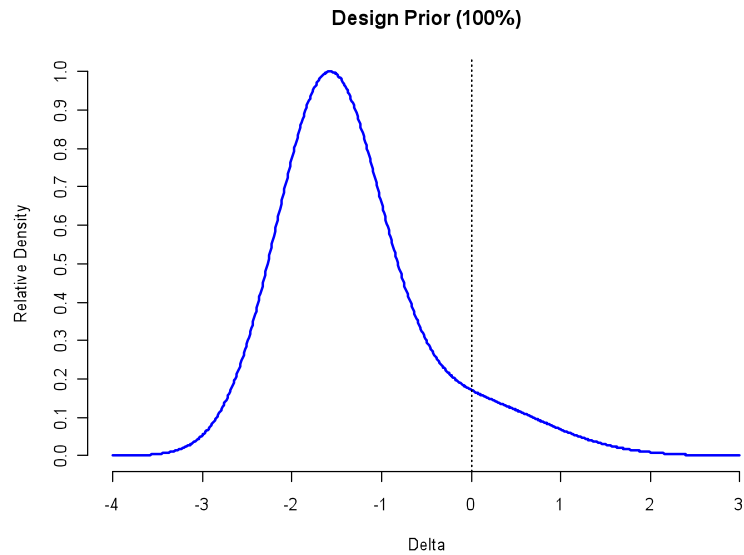


# Study OC Summaries

	Effective Cmpd $\Delta > 0.7$	Ineffective Cmpd $\Delta \leq 0.7$
Stop (Not superior to P bo)	<b>0.11</b> Pr(Stop   Eff)	<b>0.89</b> Pr(Stop   Ineff) "Specificity"
Not Stop (Superior to P bo)	<b>0.89</b>	<b>0.11</b>
Total	1.0	1.0

	Effective Cmpd $\Delta > Nx$	Ineffective Cmpd $\Delta \leq Nx$
Not PoC (Not superior to Nx)	<b>0.42</b>	<b>0.99</b>
PoC (Superior to Nx)	<b>0.58</b> Pr(Go   Eff) "Sensitivity"	<b>0.01</b> Pr(Go   Ineff)
Total	1.0	1.0

# Positive & Negative Predictive Distributions

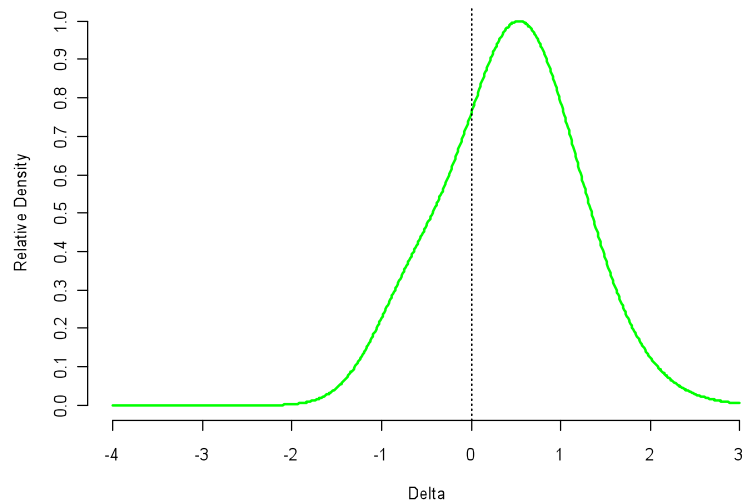


% Correct(95) / Sens,Spec

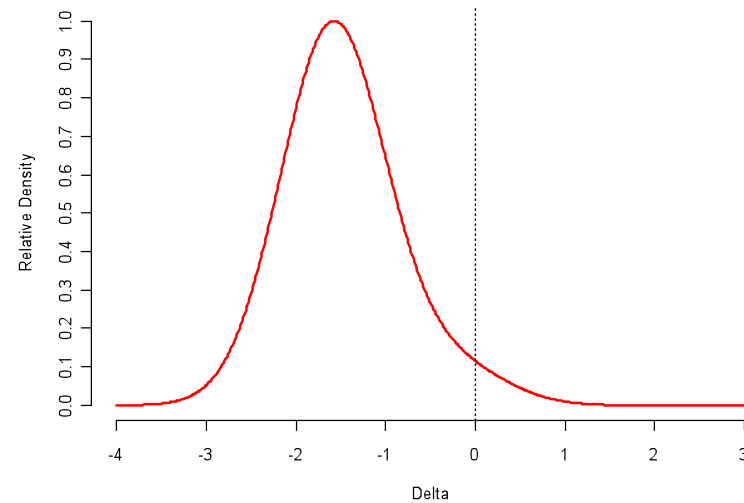
GO(6 / 66)

NOT GO(89 / 97)

Predictive Posterior | GO(8%)



Predictive Posterior | NOT GO(92%)



# Interim Analysis

Stop for futility

Probability of C1 at study end  $< 20\%$ .

Stop for clear efficacy

Probability of C2 at study end  $> 80\%$ .

Continue otherwise.

# Practical Issues & Value of Bayesian Design

## Education & training in Bayesian approach

Philosophy as well as methods

Elicitation: purpose & “how to do it”

Software tools for non-experts

e.g. BugsXLA GUI for WinBUGS, R Scripts

## Precedented methods not available

What do we do with all the new probabilities?

Classical :  $p(\text{Pass test} \mid \text{true value})$  : power

Bayesian:  $p(\text{Pass test})$  :  $E(\text{power})$

$p(\text{true value} \mid \text{Pass test})$  : +ve pred. post.

$p(\text{true value})$  : prior